1 Intro

After evaluating the problem and taking into consideration all the given data, along with the TSD (Technical Sheet Data) found for the two airplanes, namely the Cessna 152 Skyhawk ¹ and the Dash 8-400 ², and after considering the state-of-the-art Lithium-metal battery ³ produced by Cuberg the current year (2022), we believe Company X's claims are not realistic.

An overview of the different identified problems is hereby given, along with the actual mathematical correct values calculated.

2 Method

The comparison between a regular Cessna 152 Skyhawk and its respective electrical version - as stated by Company X - along with the DATA used is then given in table 1. Several assumptions have been taken for the calculations.

A flight distance of 700 km and a pilot weight of 80 kg have been assumed for both airplanes. Furthermore, 80 passengers have been taken into account with the same average weight for the DASH 8-400, as we thought it would make sense that if one flies an airplane of that size, one should consider the fact that passengers and/or cargo should be carried: there would be no point - other than a demonstration of technological improvement - to fly an empty DASH.

We estimated that a Cessna Skyhawk would consume around 85 kg of fuel for the stated distance which, assuming a fuel energy density of $12.1 \ kWh/kg$, corresponds to an energy consumption of $1024 \ kWh$. For the analysis of the electric Cessna, the max ramp weight has been considered, and it's been assumed that all the remaining "usable" weight is taken up by the battery, thus de facto considering the electric motor to be weightless. Another issue that should be considered following Company X's claims is the volume that these components would occupy and their placement inside the plane. Given that the jet motor and fuel tank are still in the plane, there is little space where the battery packs could be placed, and considering their substantial weight in relation to the weight of the plane itself, this could affect both energy consumption, structural integrity, balance at take-off and landing, rate of climb, speed and controllability.

According to these assumptions, the maximum weight of the battery is 270 kg. For the same 700 km long trip, and corresponding energy consumption of 1027 kWh (taking into account the slightly larger total mass in this case), the used battery would need to have an energy density of 3.8 kWh, which is about one order of magnitude larger than the current state of technology of about 0.4 kWh/kg.

Considering now a battery of 270 kg with an energy density equal to the current state-of-the-art lithium metal battery, claimed to be 405 Wh/kg, the maximum possible travel distance would be roughly 73 km. Variations in energy consumption depending on speed have not been taken into consideration. To deliver the energy needed for a 700 km long trip, such a battery would need to weigh over 2500 kg, which would in turn drastically increase

 $^{^1}https://cessna.txtav.com/en/piston/cessna-skyhawk \\$

 $^{^2} https://www.airlines-inform.com/commercial-aircraft/dash-8q400.html\\$

 $^{^3}https://cuberg.net/news/cuberg-lithium-metal-external-cell-cycle-life-validation$

the needed energy, not to mention that it's well over the weight of the plane itself.

Table 1: DATA for Cessna 152 Skyhawk and electric plane claimed by Company X. Empty Plane Weight (m_{plane}^{empty}) , Max Ramp Weight (m_{ramp}) , Fuel Density (ρ_{fuel}) , Carried Fuel Mass (m_{fuel}) , Max battery weight constrained by Ramp weight $(m_{battery}^{max})$, Energy density of Battery required for the 700 km trip (u), "/" means that the relative value is not given for respective plane.

	$m_{plane}^{empty}[Kg]$	$m_{ramp}[Kg]$	$\rho_{fuel}[kWh/Kg]$	$m_{fuel}[Kg]$	$m_{battery}^{max}[Kg]$	u[kWh/Kg]
Cessna 152 SkyHawk	762	1160	12,1	85	/	/
Electric Cessna	762	1160	/	48	270	1,1
Dash8-Q400	17090	28000	12,1	1827	/	/
Electric Dash	17090	28000	/	1772	2745	2,4

Following a similar reasoning applied to a Dash 8-400, the necessary energy density of the battery would have to be 8 kWh/kg, that is 20 times greater than the current state-of-the-art technology. From this data it can be observed that a 20 fold increase in ramp weight (28000 Kg) corresponds to 10 times higher battery weight, doubling the required energy density of the battery.

These results make sense when considering that the necessary speed of the plane increases with increasing weight, as the drag force from equation 1, further expanded in equation 2, is proportional to the velocity squared. This in turn shows that the power is proportional to the velocity cubed (as shown in equation 3): meaning that the problem is even greater than what initially assumed, since in our calculations a linear relationship between the two has been considered for simplicity reasons.

$$P = F_D * v \tag{1}$$

$$F_D = \frac{1}{2}\rho \cdot v^2 \cdot C_D \cdot A \tag{2}$$

$$P \propto v^3 \tag{3}$$

 F_D being the drag force, ρ being the density of the fuel, C_D being the drag coefficient, A being the reference area and v the flow velocity relative to the airplane.

3 Conclusion

Though electric plane flight is possible and has been achieved for short distances and times, the range that such machines can cover is very limited and not suited for commercial flight at the current state of the technology. We however believe that many solutions could be adopted to ease the transition towards green flights (i.e. Biofuel), but such solutions include using completely different technologies with their respective different problems; hence we thought it was not useful to provide any further advice on the matter. In conclusion we all agree on the fact that nobody should invest in Company X as what stated is most likely to be false.